Core Mathematics C2 Paper B

1. A sequence is defined by

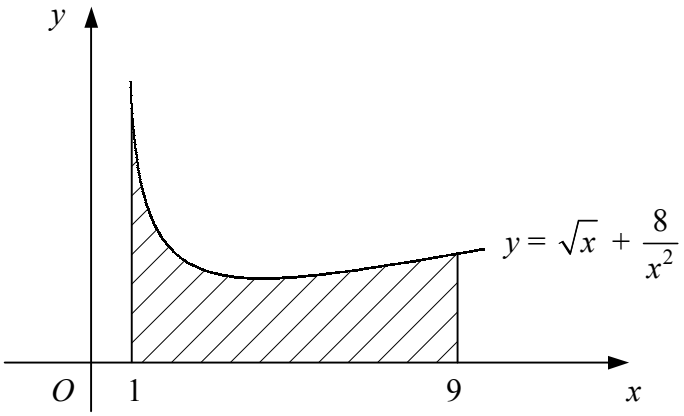
$$u_{n+1} = \frac{u_n + 1}{3}, \quad n = 1, 2, 3, \ldots$$

Given that $u_3 = 5$,

(i) find the value of u_4, [1]

(ii) find the value of u_1. [3]

2.



The diagram shows the curve with equation $y = \sqrt{x} + \frac{8}{x^2}, \quad x > 0$.

Show that the area of the shaded region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 9$ is $24\frac{4}{9}$. [5]

3. *(i)* Show that the equation

$$3\cos^2 x° + \sin^2 x° + 5\sin x° = 0$$

can be written as a quadratic equation in $\sin x°$. [2]

(ii) Hence solve, for $0 \leq x < 360$, the equation

$$3\cos^2 x° + \sin^2 x° + 5\sin x° = 0.$$ [5]

4. *(a)* Sketch the curve $y = 5^{x-1}$, showing the coordinates of any points of intersection with the coordinate axes. [2]

(b) Find, to 3 significant figures, the x-coordinates of the points where the curve $y = 5^{x-1}$ intersects

(i) the straight line $y = 10$, [3]

(ii) the curve $y = 2^x$. [3]

5. As part of a new training programme, Habib decides to do sit-ups every day.

He plans to do 20 per day in the first week, 22 per day in the second week, 24 per day in the third week and so on, increasing the daily number of sit-ups by two at the start of each week.

(i) Find the number of sit-ups that Habib will do in the fifth week. [3]

(ii) Show that he will do a total of 1512 sit-ups during the first eight weeks. [2]

In the nth week of training, the number of sit-ups that Habib does is greater than 300 for the first time.

(iii) Find the value of n. [3]

6. *(i)* Write down the exact value of $\cos \frac{\pi}{6}$. [1]

The finite region R is bounded by the curve $y = \cos^2 x$, where x is measured in radians, the positive coordinate axes and the line $x = \frac{\pi}{3}$.

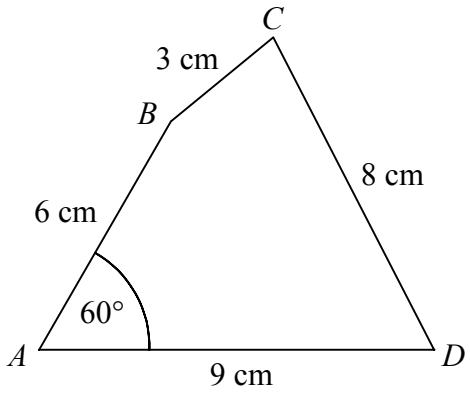
(ii) Use the trapezium rule with two intervals of equal width to estimate the area of R, giving your answer to 3 significant figures. [4]

The finite region S is bounded by the curve $y = \sin^2 x$, where x is measured in radians, the positive coordinate axes and the line $x = \frac{\pi}{3}$.

(iii) Using your answer to part *(b)*, find an estimate for the area of S. [3]

Turn over

7.



The diagram shows the quadrilateral $ABCD$ in which $AB = 6$ cm, $BC = 3$ cm, $CD = 8$ cm, $AD = 9$ cm and $\angle BAD = 60°$.

(i) Show that $BD = 3\sqrt{7}$ cm. [3]

(ii) Find the size of $\angle BCD$ in degrees to 1 decimal place. [3]

(iii) Find the area of quadrilateral $ABCD$. [3]

8. $\mathrm{p}(x) = x^4 - (x - 2)^4$.

(i) Show that $(x - 1)$ is a factor of $\mathrm{p}(x)$. [2]

(ii) Show that

$$\mathrm{p}(x) = 8x^3 - 24x + 32x - 16.$$ [4]

(iii) Find the quotient and remainder when $\mathrm{p}(x)$ is divided by $(x + 1)$. [4]

9. The curve C has the equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}'(x) = 1 + \frac{2}{\sqrt{x}}, \quad x > 0.$$

The straight line l has the equation $y = 2x - 1$ and is a tangent to C at the point P.

(i) State the gradient of C at P. [1]

(ii) Find the x-coordinate of P. [3]

(iii) Find an equation for C. [6]

(iv) Show that C crosses the x-axis at the point $(1, 0)$ and at no other point. [3]